



Machine learning assisted inverse design of microresonators

ARGHADEEP PAL,^{1,2} ALEKHYA GHOSH,^{1,2} SHUANGYOU ZHANG,¹
TOBY BI,^{1,2}  AND PASCAL DEL'HAYE^{1,2,*}

¹Max Planck Institute for the Science of Light, 91058 Erlangen, Germany

²Department of Physics, Friedrich-Alexander-Universität Erlangen-Nürnberg, 91058 Erlangen, Germany

*pascal.delhaye@mpl.mpg.de

Abstract: The high demand for fabricating microresonators with desired optical properties has led to various techniques to optimize geometries, mode structures, nonlinearities, and dispersion. Depending on applications, the dispersion in such resonators counters their optical nonlinearities and influences the intracavity optical dynamics. In this paper, we demonstrate the use of a machine learning (ML) algorithm as a tool to determine the geometry of microresonators from their dispersion profiles. The training dataset with ~460 samples is generated by finite element simulations and the model is experimentally verified using integrated silicon nitride microresonators. Two ML algorithms are compared along with suitable hyperparameter tuning, out of which Random Forest yields the best results. The average error on the simulated data is well below 15%.

© 2023 Optica Publishing Group under the terms of the [Optica Open Access Publishing Agreement](#)

1. Introduction

The small mode volumes and high quality (Q) factors of microresonators have made them excellent tools to confine light and steer up high nonlinear effects with low input power threshold [1]. One example is four-wave mixing (FWM), which leads to the formation of optical frequency combs [2]. These combs have shown great applications [3], mainly in the fields of spectroscopy [4], sensing [5], telecommunications [6], and waveform generation [7], due to their broad spectrum with uniform spacings. Sufficiently good overlaps of the equidistant FWM generated sidebands and the respective resonance frequencies of the microresonator are needed to generate broadband frequency combs. Thus, the comb formation is strongly influenced by dispersion, which leads to uneven spacings between the resonance frequencies. These deviations in resonances from the equidistant positions are evaluated via the integrated dispersion D_{int} , which can be written as

$$D_{\text{int}}(m) = \omega_m - \omega_0 - D_1 \times (m - m_0). \quad (1)$$

Here, ω_m is the angular resonance frequency of the m^{th} mode with respect to the pump mode m_0 at angular pump frequency ω_0 . $D_1/2\pi$ is the free spectral range (FSR) around the pump mode. Resonator dispersion also plays an important role in the temporal intracavity soliton dynamics. Out of many applications, the generation of bright Kerr frequency combs occurs in presence of anomalous dispersion [8], whereas, normal dispersion [9] favors the formation of dark pulse solitons as shown in Fig. 1(a), where D_{int} has been plotted as a function of wavelength for silicon nitride (Si_3N_4) ring resonators with similar radii but different core heights and widths. Recently, it has been demonstrated that dark-bright soliton bound states can be generated in a microresonator crossing different dispersion regimes [10], leading to light states with close to constant output power but resembling a comb in the frequency domain. In addition, resonators with zero dispersion not only exhibit different soliton dynamics, but also are desirable for ultra-broadband comb generation [11,12]. Therefore, proper engineering of the dispersion becomes vital [13,14]. Dispersion calculations can be carried out via simulations and experiments. A quick analysis of dispersion is not possible by the conventional simulation techniques due to their iterative

nature leading to high time-complexity. Most importantly, this simulation or measurement is a unidirectional process, i.e., from the structure, one can get an idea about the dispersion but not the other way around. Methods of the inverse design of microresonators [15–17] usually follow the approach of starting with a structure from empirical knowledge and then iteratively modifying the geometries based on the deviation from the desired dispersion profile. Moreover, some inverse design methods [18] require additional quality factor engineering. Because of the highly nonlinear relation between the geometrical specifications of a microresonator structure and its dispersion profile, a simple method like linear regression for the generation of structures is not feasible. Advanced optimization methods such as evolutionary algorithms can help us in

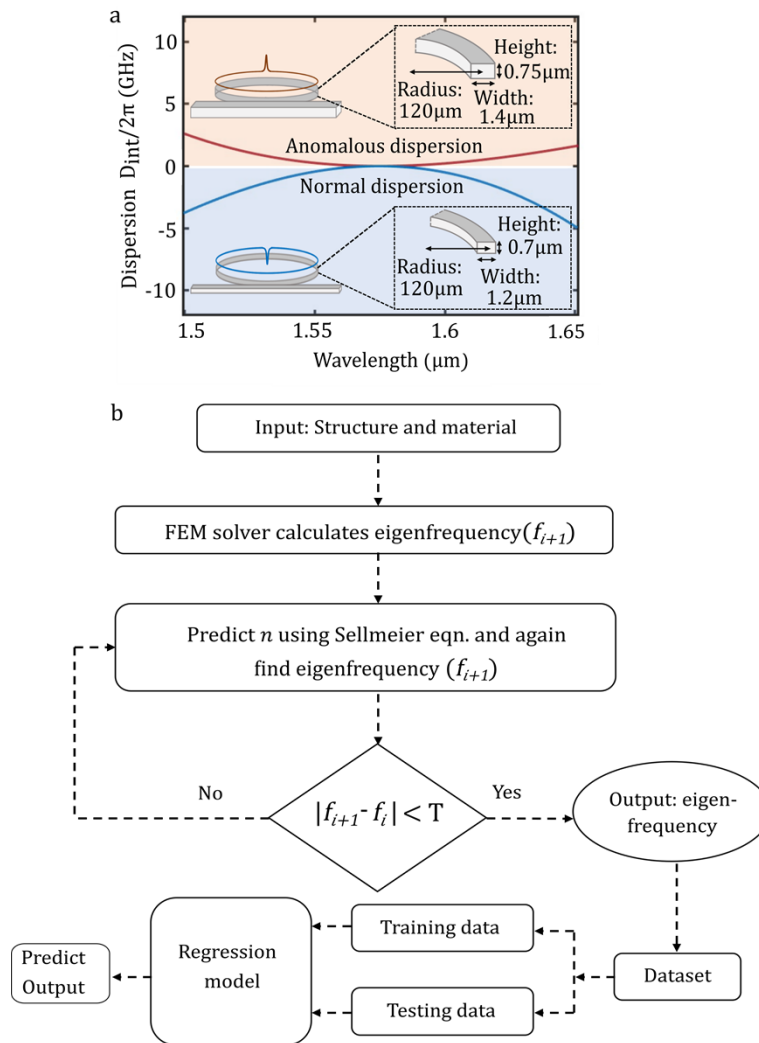


Fig. 1. (a) Influence of microresonator geometry on the integrated dispersion. Inset: The structural dimensions of the resonator core cross-section are shown. (b) Flowchart for simulating the resonance frequencies for a given structure and material. After completion of this iterative process, the data is used to calculate the dispersion parameter. Part of the dataset is used to train the regression model and the rest is used to test the prediction of the model.

that aspect, however, extensive numerical simulations are required in each step of the process, making the process slow and time-consuming [19].

Machine learning (ML) is highly successful to find inherently complex nonlinear relationships between datasets in domains like, signal processing [20], natural language processing [21], computer vision [22] and data mining [23]. Due to the vast success, applications of ML transcend the boundaries of computer science to fields like classical and quantum photonics as well [24,25]. The propagation of pulses through a nonlinear medium has been effectively modelled by a recurrent neural network [26]. ML algorithms have been utilized for predicting dispersion relations of photonic crystals [27], estimating effective refractive index, mode area, dispersion and other parameters of photonic crystal fibers [28], as well as for developing photonic crystal based gas sensors [29]. In the problem of photonic inverse designing, various ML approaches show impressive efficiencies [30]. A deep learning (DL) based bidirectional model has successfully predicted subwavelength geometry of nanoparticles from far-field optical response [19]. For a desired scattering profile, the geometries of the nanoparticles have been predicted using an artificial neural network [31]. A generative adversarial network based model has been used to predict metasurface geometries [32].

In this work, we propose a novel method to predict the structures of the microresonators based on the dispersion parameter D_{int} using ML. For the inverse design, a regression ML model has been trained by creating a dataset simulating dispersion variation over wavelength for different microresonator geometries. The simulations have been carried out using the finite element method (FEM) in COMSOL. After the validation of the model with the test data, this ML technique is used to estimate the heights, widths, and radii of the fabricated resonators from the experimentally obtained dispersion plots. Among different dispersion measurement techniques available [13], we chose frequency comb assisted diode laser spectroscopy [33].

2. Experimental procedures and simulations

The dispersion simulations are performed with structural and material parameters as input as shown in Fig. 1(b). To initiate the FEM solver, an approximate resonance frequency is needed which is given by

$$f_{\text{guess}} = \frac{mc}{2\pi Rn}, \quad (2)$$

where c is the speed of light, m is the mode number, n is the refractive index of the core and R being the radius of the resonator. The output of the solver gives the eigenfrequency which is used to get an idea of the actual refractive index of the medium for that particular optical frequency using the Sellmeier equation for the material of the waveguide [13]. The Sellmeier equation for the Si_3N_4 core is

$$n^2(\lambda) = 1 + \frac{3.0249\lambda^2}{\lambda^2 - 0.1353406^2} + \frac{40314\lambda^2}{\lambda^2 - 1239.842^2}, \quad (3)$$

where λ is the wavelength in micrometer. This iterative process continues until the difference between the resonance frequencies obtained from two consecutive iterations lies within a certain threshold (T). With an increase in the size of the resonator, the number of modes within a fixed spectral range increases, thereby increasing the simulation time. From the resonance frequencies obtained from the simulation, the dispersion parameter D_{int} is calculated. In this work, we use Si_3N_4 as resonator material. Introducing an ML model requires a dataset for the purpose of training and testing. For the generation of the dataset for the Si_3N_4 ring resonators, the radii of the rings are varied from 30 μm to 130 μm at an interval of 20 μm , whereas, the core widths are chosen from 1 μm to 2 μm in steps of 0.1 μm . Similarly, equally spaced height values are considered from 500 nm to 800 nm in steps of 50 nm. The cladding considered here is made of fused silica and has a width and height of 4 μm each. These ranges of parameters for the simulated

structures were chosen around the values that can be fabricated in our lab. The simulated dataset of 462 samples contains resonance wavelength in the range between 1 μm and 2 μm .

The variation of FSR over frequency is illustrated in Fig. 2(a) for both normal and anomalous dispersion regime. To verify the regression model, trained from the simulation dataset, it is important to check whether comparable structural details are acquired when dispersion parameters obtained from experiments are fed as input. Figure 2(b) shows the experimental setup to collect the transmission spectrum as well as the calibration point for dispersion estimation [33]. The SEM image for the 50- μm -radius resonator is portrayed in Fig. 2(c). The external diode cavity laser (ECDL) is swept from 1520 nm to 1615 nm and a microresonator transmission spectrum is recorded by an oscilloscope. Simultaneously, frequency calibration points are obtained by beating the sweeping diode laser with a stable commercial frequency comb (repetition rate 100 MHz), filtered through two bandpass filters (central frequencies at 35 MHz and 45 MHz with 2% and 1% filter bandwidths respectively). The frequency calibration points are simultaneously recorded by the oscilloscope.

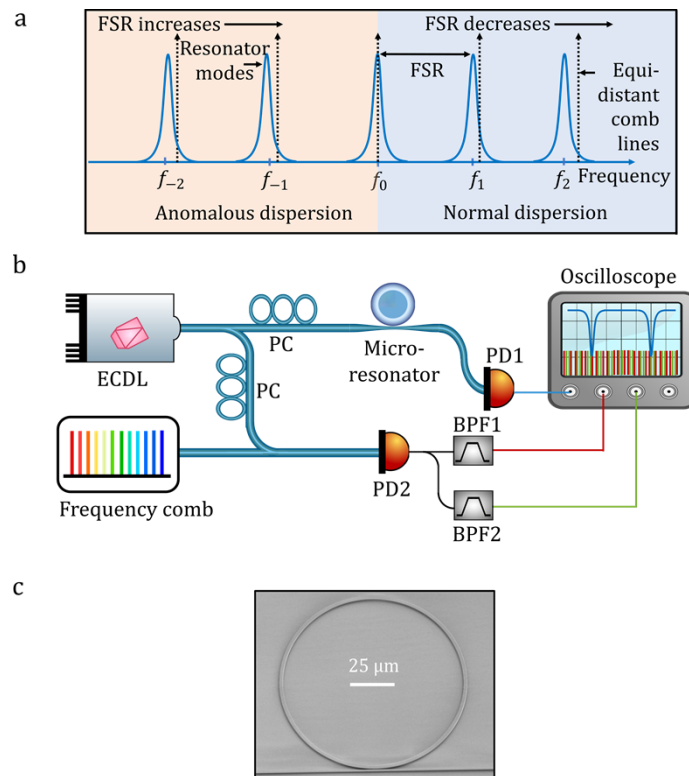


Fig. 2. (a) Variation of FSR with frequency for different kinds of dispersion. (b) The experimental setup used to measure the dispersion of the various microresonators. ECDL: External cavity diode laser, PC: Polarization controller, PD: Photodiode, BPF: Bandpass filter. (c) Scanning electron microscope (SEM) image of the 50- μm -radius resonator.

3. Results and discussions

3.1. Simulation results

The performance of the geometrical parameter prediction model on the simulated dataset using Random Forest (RF) and Decision Tree (DT) regressors are presented in this section. Given

the small size of the dataset, ML models have been chosen over DL models. The RF model has empirically outperformed other ML algorithms such as support vector machine [34] and artificial neural networks [35]. Both RF and DT algorithms have been implemented using the “Scikit-learn” library in python. The performances are evaluated in terms of mean absolute percentage error (MAPE). It is defined as,

$$\text{MAPE} = \frac{100\%}{n} \sum_{i=1}^n \left| \frac{x_i^{\text{pred}} - x_i^{\text{act}}}{x_i^{\text{act}}} \right|, \quad (4)$$

where x_i^{act} and x_i^{pred} are target and predicted values for i^{th} sample respectively. In each case, the dataset has been divided randomly into two sections, one section (75% of the total samples) is used for training the model and the rest (25% of the total samples) for testing. Both DT and RF have hyperparameters, which control the extent of nonlinearity that can be modelled by the used architecture. In a DT regression model or a regression tree, starting from the root nodes, the dataset is split at each internal node depending upon the feature values. The condition of splitting in each node is decided by minimizing the root mean square error between the samples and the node threshold. We can choose the method of splitting at each node, the maximum depth of a tree and the minimum number of samples in a leaf node. An RF model consists of many DTs. From the training dataset, a section is selected with replacement for each of the trees. For each DT, parallel data fitting occurs and, in the end, an average of all the tree predictions is used as the final output of the RF model. In this case, the number of trees, amount of data used to build each tree, etc. are the hyperparameters that can be tuned to achieve better performance. In this work, we tuned the regressors over many possible combinations of the hyperparameters and selected the set of values that gives the best performance, a process known as grid search. In the grid search for RF, the number of estimators was varied from 20 to 230 and the maximum depth was varied from 5 to 100. For DT, the maximum depth was varied from 5 to 100 and the minimum samples leaf was varied from 1 to 10. Table 1 presents the best hyperparameters for the two ML regressors and their corresponding performances.

Table 1. Performance of regressors for inverse design

Name of Model	DT	RF	RF
Data type	Not normalized	Not normalized	Normalized
Best model Hyper-parameters	Max. depth: 12	Max. depth: 20	Max. depth: 80
	Min. samples leaf: 7	Estimators: 200	Estimators: 180
MAPE in radius pred. (%)	27.16	12.52	21.43
MAPE in height pred. (%)	9.15	3.14	1.89
MAPE in width pred. (%)	22.68	15.01	6.34

It can be seen that the DT model yields a consistent yet poor prediction accuracy. However, when using RF, the performance improves drastically. The optimal model of RF regressor with not normalized data has 200 different DTs and can be seen that the MAPE for all the structural parameters are less than or around 15% for the worst case. Normalization of the dataset affects the performance of ML algorithms. Therefore, we performed the training step for RF regressor again with max-min normalized data and the results are presented in Table 1. It can be observed that, the hyperparameters of the RF model are different for normalized and not-normalized data samples, as for the two cases we have performed the grid search separately and noted the best set of hyperparameters in each case. From a comparison of the columns in Table 1, it can be seen that normalization of input and output dataset deteriorates the performance of the RF model in terms of radius prediction. Error in radius affects the result significantly, since change in radius affects

the FSR of the resonator itself notably. One of the main concerns for our model is that overfitting may affect the performance of the model due to the small size of the training data. Overfitting is encountered when a regressor increases nonlinearity to a great extent such that it fits to all of the datapoints in the training dataset, thereby reducing the performance on the test dataset. We followed cross-validation to tackle this problem. 4-fold cross-validation has been used for all of the above regressors. In this process, the total training dataset is divided into four parts in each training step, the training is done with three parts and the model is validated with the remaining section. Figure 3 shows a histogram that illustrates the overall performance of the prediction model on the test-segment of the simulated data. The performance of the model on the test-data shows that the median of the errors in all three design parameters are less or around 10%. In the inset, it can be seen that with increasing number of trees in an RF model, the negative mean absolute error (NMAE) decreases initially, but saturates afterwards. The performance of our model has motivated us to validate the model using experimental data. Therefore, we have applied the ML model, trained on simulated data, to predict structural parameters from experimental measurements. In order to check the effect of the prediction error (on average 10%) on the resulting dispersion, we calculated the change in dispersion induced by increasing each structural parameter (radius, height, width) separately by 10%. The corresponding MAPE for the integrated dispersion across 1500 nm to 1600 nm (experimental range) are found to be 0.01%, 0.04% and 0.05% respectively, which is not very significant. The simulation for dataset generation has been carried on a computer with an Intel Xeon W-1290P CPU @ 3.70 GHz and 64 GB RAM. For resonators of radii 150 μm . and 50 μm , the elapsed times for the simulation of eigen frequencies between 1 μm and 2 μm are around 197 minutes and 71 minutes, respectively. In evolutionary approaches of optimization, which may be considered as an advanced optimization method, huge number of simulations (often more than hundreds) are required for every structure design task [19]. ML significantly speeds up the prediction of structural parameters, independent of the resonator size. A trained RF model (with specifications: max. depth = 20, estimators = 200) predicts structure from a dispersion profile in an average of 0.013 seconds.

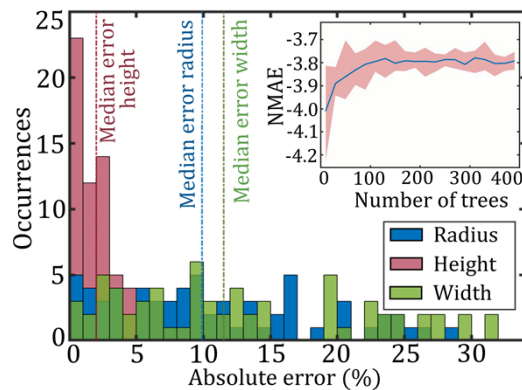


Fig. 3. Histogram showing the prediction performance of the optimized RF model on simulated data. The dashed lines indicate the median values of prediction error for each parameter. Inset: Negative mean absolute error of the RF model versus number of trees in the model.

3.2. Experimental results

To validate the accuracy of our prediction model, we fabricated two Si_3N_4 ring microresonators, measured their dispersion, then fed the dispersion values to the RF model and finally, compared the predicted structural details with the original designs. The resonators under test have different

radii and waveguide heights but same waveguide width. The transmission profiles of these two resonators with radii of $50\ \mu\text{m}$ and $100\ \mu\text{m}$ are shown in Fig. 4(a) and 4(c) respectively, along with their corresponding D_{int} plots in Fig. 4(b) and 4(d). The pump wavelength is considered to be around $1557\ \text{nm}$. Since the transmission plots contain multiple mode families, the one with higher Q-factor has been used for dispersion measurement in each case. The resonators have loaded Q-factors around 5×10^5 (around $0.3\ \text{GHz}$ linewidth). The dimensions of the actual and the predicted structures for both experiments have been listed in Fig. 4(e) and 4(f). The simulated dispersion profiles for both the predicted structure and the SEM observed structure are also portrayed in both Fig. 4(b) and 4(d). In the second case, the height of the microresonator is chosen to be slightly outside the training range to check whether the model can still predict in this range. The model predicted the radius, height and width within the 15% error margin. The MAPE error for the experimental case is similar to that found in Table 1 for the simulated training dataset.

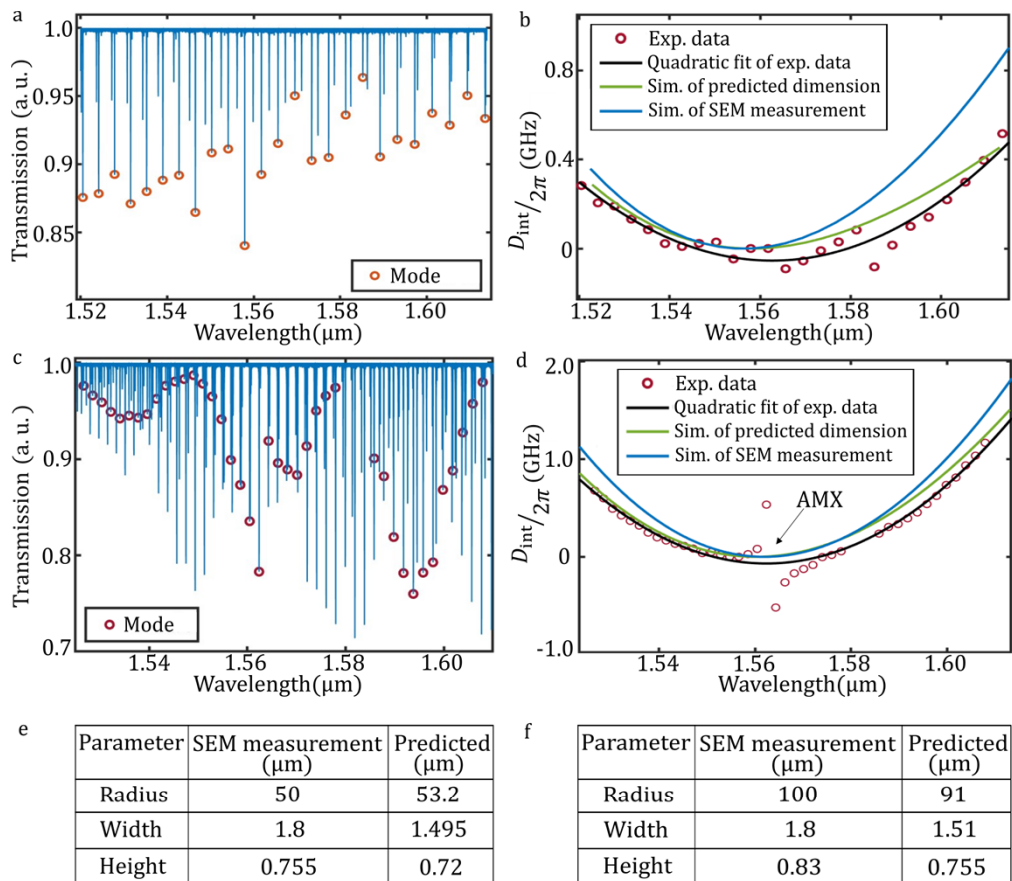


Fig. 4. Microresonator resonance spectra and corresponding dispersions used to predict the structural parameters of the microresonator. (a) Transmission spectrum of a $50\text{-}\mu\text{m}$ -radius Si_3N_4 resonator with its corresponding fundamental mode family being highlighted. (b) Integrated dispersion for the fundamental mode along with a quadratic fit for the spectrum in panel (a). (c), (d) The spectrum and dispersion for a $100\text{-}\mu\text{m}$ -radius resonator. The simulation of the predicted dimensions and the SEM observed dimensions (after fabrication) have been plotted in both (b) and (d). (e), (f) Dimensions of the resonators predicted by the RF model from the experimentally obtained integrated dispersion values and comparison to the actual structures.

There are fabrication uncertainties regarding the accuracy of the structural parameters (around 10 nm for the best case) and also there can be additional impurities or inclinations of the core walls, which alter the dispersion curve and can contribute to the deviation between the actual and predicted geometrical parameters. In some cases, different resonator geometries can have similar dispersion profiles. Moreover, the avoided mode crossing (AMX) [36], as seen prominently in Fig. 4(d) around 1560 nm, changes slightly the quadratic fit of the dispersion profile. The sensitive ML models are influenced by such minute deviations. These kinds of crossings are not encountered during the simulations as the walls of the core are considered to be exactly perpendicular which is very difficult to maintain during fabrication of the resonators. The effect of these crossings can be negated if the FSR is small or the scanning range is broader so that we can have more and more modes and thus the fitting becomes easier. More importantly, if the dataset can be increased along with decreasing the intervals between the chosen structural parameters, the prediction the proposed ML model can improve, additionally opening the possibilities of application of DL models for the discussed inverse design problem.

4. Conclusion and outlook

We have trained a machine learning model on simulated data for predicting the structure of integrated ring resonators from their dispersion profiles generated by varying the height, radius and width of microresonators in finite element simulations. Comparing “decision tree” and “random forest” algorithms, we concluded that “random forest” yields better performance for the generation of microresonator structures. The optimized and tuned model has been used for structure prediction based on experimentally measured dispersion profiles from silicon nitride resonators. Increasing the size and spectral width of the training dataset and subsequent applications of DL could further increase the accuracy of the inverse design performance. With the ever-growing possibilities in fabrication of photonic integrated circuits, machine learning can play an important role to precisely engineer dispersion for a large number of different applications. In particular, this enables the generation of spectrally tailored optical soliton frequency combs in chip-integrated microresonators. In the future, this work can be extended to more complex geometries with organically changing resonator dimensions, which enables even more broadband control of microresonator mode structures.

Funding. Max-Planck-Gesellschaft; H2020 Marie Skłodowska-Curie Actions (812818); H2020 European Research Council (756966).

Acknowledgment. We thank Olga Lohse, Irina Harder, Florentina Gannott, and Alexander Gumann from the Technology Development & Service Units (TDSU1) of the Max Planck Institute for the Science of Light for their support with the microresonator fabrication process.

Disclosures. The authors declare no conflicts of interest.

Data availability. Data from this article are available from the authors upon reasonable request.

References

1. T. J. Kippenberg, S. M. Spillane, and K. J. Vahala, “Kerr-Nonlinearity Optical Parametric Oscillation in an Ultrahigh- Q Toroid Microcavity,” *Phys. Rev. Lett.* **93**(8), 083904 (2004).
2. P. Del’Haye, A. Schliesser, O. Arcizet, T. Wilken, R. Holzwarth, and T. J. Kippenberg, “Optical frequency comb generation from a monolithic microresonator,” *Nature* **450**(7173), 1214–1217 (2007).
3. T. J. Kippenberg, R. Holzwarth, and S. A. Diddams, “Microresonator-Based Optical Frequency Combs,” *Science* **332**(6029), 555–559 (2011).
4. M. Yu, Y. Okawachi, A. G. Griffith, M. Lipson, and A. L. Gaeta, “Microresonator-based high-resolution gas spectroscopy,” *Opt. Lett.* **42**(21), 4442–4445 (2017).
5. P. Trocha, M. Karpov, D. Ganin, M. H. P. Pfeiffer, A. Kordts, S. Wolf, J. Krockenberger, P. Marin-Palomo, C. Weimann, S. Randel, W. Freude, T. J. Kippenberg, and C. Koos, “Ultrafast optical ranging using microresonator soliton frequency combs,” *Science* **359**(6378), 887–891 (2018).
6. N. Alic, “Frequency Combs in Telecommunications Applications,” in *Frontiers in Optics* (Optica Publishing Group, 2014), paper FM3C.6.

7. S. Zhang, J. M. Silver, X. Shang, L. D. Bino, N. M. Ridler, and P. Del'Haye, "Terahertz wave generation using a soliton microcomb," *Opt. Express* **27**(24), 35257–35266 (2019).
8. T. Herr, K. Hartinger, J. Riemensberger, C. Y. Wang, E. Gavartin, R. Holzwarth, M. L. Gorodetsky, and T. J. Kippenberg, "Universal formation dynamics and noise of Kerr-frequency combs in microresonators," *Nat. Photonics* **6**(7), 480–487 (2012).
9. X. Xue, Y. Xuan, Y. Liu, P.-H. Wang, S. Chen, J. Wang, D. E. Leaird, M. Qi, and A. M. Weiner, "Mode-locked dark pulse Kerr combs in normal-dispersion microresonators," *Nat. Photonics* **9**(9), 594–600 (2015).
10. S. Zhang, T. Bi, G. N. Ghalanos, N. P. Moroney, L. Del Bino, and P. Del'Haye, "Dark-Bright Soliton Bound States in a Microresonator," *Phys. Rev. Lett.* **128**(3), 033901 (2022).
11. M. H. Anderson, W. Weng, G. Lihachev, A. Tikan, J. Liu, and T. J. Kippenberg, "Zero dispersion Kerr solitons in optical microresonators," *Nat. Commun.* **13**(1), 4764 (2022).
12. S. Zhang, T. Bi, and P. Del'Haye, "Microresonator Soliton Frequency Combs in the Zero-Dispersion Regime," *arXiv*, arXiv:2204.02383 (2022).
13. S. Fujii and T. Tanabe, "Dispersion engineering and measurement of whispering gallery mode microresonator for Kerr frequency comb generation," *Nanophotonics* **9**(5), 1087–1104 (2020).
14. K. Y. Yang, K. Beha, D. C. Cole, X. Yi, P. Del'Haye, H. Lee, J. Li, D. Y. Oh, S. A. Diddams, S. B. Papp, and K. J. Vahala, "Broadband dispersion-engineered microresonator on a chip," *Nat. Photonics* **10**(5), 316–320 (2016).
15. G. H. Ahn, K. Y. Yang, J. Skarda, and J. Vučković, "Inverse design of microresonator dispersion for nonlinear optics," in *Conference on Lasers and Electro-Optics* (Optica Publishing Group, 2020), paper SM3L.4.
16. E. Lucas, S.-P. Yu, G. H. Ahn, K. Yang, J. Vuckovic, and S. B. Papp, "Inverse spectral design of Kerr microcomb pulses," *Proc. SPIE* **11672**, 2 (2021).
17. K. Wang, X. Ren, W. Chang, L. Lu, D. Liu, and M. Zhang, "Inverse design of digital nanophotonic devices using the adjoint method," *Photonics Res.* **8**(4), 528–533 (2020).
18. G. H. Ahn, K. Y. Yang, R. Trivedi, A. D. White, L. Su, J. Skarda, and J. Vučković, "Photonic Inverse Design of On-Chip Microresonators," *ACS Photonics* **9**(6), 1875–1881 (2022).
19. I. Malkiel, M. Mrejen, A. Nagler, U. Arieli, L. Wolf, and H. Suchowski, "Plasmonic nanostructure design and characterization via Deep Learning," *Light: Sci. Appl.* **7**(1), 60 (2018).
20. H. Purwins, B. Li, T. Virtanen, J. Schlüter, S.-Y. Chang, and T. Sainath, "Deep Learning for Audio Signal Processing," *IEEE J. Sel. Top. Signal Process.* **13**(2), 206–219 (2019).
21. P. M. Nadkarni, L. Ohno-Machado, and W. W. Chapman, "Natural language processing: an introduction," *J. Am. Med. Inform. Assoc.* **18**(5), 544–551 (2011).
22. A. Vouliodimos, N. Doulamis, A. Doulamis, and E. Protopapadakis, "Deep Learning for Computer Vision: A Brief Review," *Comput. Intell. Neurosci.* **2018**, 1–13 (2018).
23. X. Teng and Y. Gong, "Research on Application of Machine Learning in Data Mining," *IOP Conf. Ser.: Mater. Sci. Eng.* **392**(6), 062202 (2018).
24. G. Genty, L. Salmela, J. M. Dudley, D. Brunner, A. Kokhanovskiy, S. Kobtsev, and S. K. Turitsyn, "Machine learning and applications in ultrafast photonics," *Nat. Photonics* **15**(2), 91–101 (2021).
25. Z. A. Kudyshev, V. M. Shalaev, and A. Boltasseva, "Machine Learning for Integrated Quantum Photonics," *ACS Photonics* **8**(1), 34–46 (2021).
26. L. Salmela, N. Tsiipinakis, A. Foi, C. Billet, J. M. Dudley, and G. Genty, "Predicting ultrafast nonlinear dynamics in fibre optics with a recurrent neural network," *Nat. Mach. Intell.* **3**(4), 344–354 (2021).
27. A. da Silva Ferreira, G. N. Malheiros-Silveira, and H. E. Hernández-Figueroa, "Computing Optical Properties of Photonic Crystals by Using Multilayer Perceptron and Extreme Learning Machine," *J. Lightwave Technol.* **36**(18), 4066–4073 (2018).
28. S. Chugh, A. Gulistan, S. Ghosh, and B. M. A. Rahman, "Machine learning approach for computing optical properties of a photonic crystal fiber," *Opt. Express* **27**(25), 36414–36425 (2019).
29. A. Ghosh, A. Pal, and N. R. Das, "An approach to design photonic crystal gas sensor using machine learning," *Optik* **208**, 163997 (2020).
30. W. Ma, Z. Liu, Z. A. Kudyshev, A. Boltasseva, W. Cai, and Y. Liu, "Deep learning for the design of photonic structures," *Nat. Photonics* **15**(2), 77–90 (2021).
31. J. Peurifoy, Y. Shen, L. Jing, Y. Yang, F. Cano-Renteria, B. G. DeLacy, J. D. Joannopoulos, M. Tegmark, and M. Soljačić, "Nanophotonic particle simulation and inverse design using artificial neural networks," *Sci. Adv.* **4**(6), eaar4206 (2018).
32. Z. Liu, D. Zhu, S. P. Rodrigues, K.-T. Lee, and W. Cai, "Generative Model for the Inverse Design of Metasurfaces," *Nano Lett.* **18**(10), 6570–6576 (2018).
33. P. Del'Haye, O. Arcizet, M. L. Gorodetsky, R. Holzwarth, and T. J. Kippenberg, "Frequency comb assisted diode laser spectroscopy for measurement of microcavity dispersion," *Nat. Photonics* **3**(9), 529–533 (2009).
34. L. Wang, X. Zhou, X. Zhu, Z. Dong, and W. Guo, "Estimation of biomass in wheat using random forest regression algorithm and remote sensing data," *Crop J.* **4**(3), 212–219 (2016).
35. P. Roßbach, "Neural Networks vs. Random Forests – Does it always have to be Deep Learning?" Germany: Frankfurt School of Finance and Management (2018).
36. T. Herr, V. Brasch, J. D. Jost, I. Mirgorodskiy, G. Lihachev, M. L. Gorodetsky, and T. J. Kippenberg, "Mode Spectrum and Temporal Soliton Formation in Optical Microresonators," *Phys. Rev. Lett.* **113**(12), 123901 (2014).